Paper I (Set 1) Suggested solution

1.

$$\frac{3x}{16-x^2}-\frac{5}{x-4}+\frac{2}{4+x}=\frac{7}{4}$$
$$\frac{3x}{(4-x)(4+x)}+\frac{5}{(4-x)}+\frac{2}{4+x}=\frac{7}{4}$$
$$4\left[3x+5(4+x)+2(4-x)\right]=7(4-x)(4+x)$$
$$12x+80+20x+32-8x=112-7x^2$$
$$7x^2+24x=0$$
$$x(7x+24)=0$$
$$x=0 \text{ or } x=-\frac{24}{7}$$

2.

$$4^{2x}-2(4^{x+2})+112=0$$
$$\left(4^x\right)^2-32\left(4^x\right)+112=0$$
$$\left(4^x-4\right)\left(4^x-28\right)=0$$
$$4^x=4 \text{ or } 4^x=28$$
$$x=1 \text{ or } x=\frac{\log 28}{\log 4}$$

3.

The old price of each bottle of soft drink $=\dfrac{132}{N}$

The new price of each bottle of soft drink $=\dfrac{132+4}{N+5}$

$$\frac{132}{N}-3=\frac{132+4}{N+5}$$
$$132(N+5)-3N(N+5)=136N$$
$$3N^2+19N-660=0$$
$$(N-12)(3N+55)=0$$
$$N=12 \text{ or } -\frac{55}{3} \text{ (Rejected)}$$

The old price of each bottle of soft drink $=\dfrac{132}{12}=\$11$

4. (a)

$$x^2 - 9x + 13 = -3x + c$$
$$x^2 - 6x + 13 - c = 0$$
$$\Delta = 0$$
$$6^2 - 4(1)(13 - c) = 0$$
$$36 - 52 + 4c = 0$$
$$c = 4$$

(b)

Sub $c = 4$,
$$y = -3x + 4$$
$$x^2 - 9x + 13 = -3x + 4$$
$$x^2 - 6x + 9 = 0$$
$$(x - 3)^2 = 0$$
$$x = 3$$

Sub $x = 3$,
$$y = -3(3) + 4$$
$$= -5$$

The coordinates of P is $(3, -5)$.

5. (a)

$$x + 3 > \frac{1}{3}(2x + 3)$$
$$3x + 9 > 2x + 3$$
$$x > -6$$

(b)

$\because x > -6$ and $-7 \le x \le 7$

$\therefore -6 < x \le 7$

6. (a)

$$x^2+10x+24\geq 0$$
$$(x+6)(x+4)\geq 0$$
$$x\leq -6 \text{ or } x\geq -4$$

(b)

Let $x=y^2-5y$

$$x\leq -6 \quad \text{or} \quad x\geq -4$$
$$y^2-5y\leq -6 \quad \text{or} \quad y^2-5y\geq -4$$
$$y^2-5y+6\leq 0 \quad \text{or} \quad y^2-5y+4\geq 0$$
$$(y-2)(y-3)\leq 0 \quad \text{or} \quad (y-1)(y-4)\geq 0$$
$$2\leq y\leq 3 \quad \text{or} \quad y\leq 1 \quad \text{or} \quad y\geq 4$$

7.

Let $\angle BAC=x$

$AB=CB$ (given)

$\angle BAC=\angle BCA=x$ (base $\angle$s, isos. Δ)

$\angle CBD=\angle BAC+\angle BCA$ (ext. $\angle$ of Δ)

$=x+x$

$=2x$

Join BE.

$\angle BEC=\angle CBD=2x$ ($\angle$ in alt. segment)

$\angle CBE=90°$ ($\angle$ in semi-circle)

$\angle BEC+\angle BCE+\angle CBE=180°$ ($\angle$ sum of Δ)

$$2x+x+90°=180°$$
$$x=30°$$

$\angle CBD=2(30°)=60°$

8. (a)

$$AC = \sqrt{AB^2 - BC^2}$$
$$AC^2 = AB^2 - BC^2$$
$$AC^2 + BC^2 = AB^2$$

$\angle ACB = 90°$ (Converse of Pyth. Theorem)

$\because \angle ACB = 90°$

$\therefore AB$ is a diameter. (Converse of $\angle$ in semi-circle)

The locus of *C* is a circle with *AB* as the diameter.

(b)

Let *E* be a point on *AB* so that $DE \perp AB$.

$$\frac{1}{2} \times 20 \times DE = 80$$
$$DE = 8$$

The locus of *D* is a line parallel to *AB* with a distance equal to 8 units.

(c)

Let *O* be the centre of the circle *ABC*.

$OP = \text{Radius} = \frac{1}{2} \times 20 = 10$ units

Let *R* be a point on *PQ* so that $OR \perp PQ$.

$\because OR \perp PQ$

$\therefore PR = QR$ (line from centre $\perp$ chord bisects chord)

$PR^2 + OR^2 = PO^2$ (Pyth. Theorem)

$$PR^2 + 8^2 = 10^2$$

$PR = 6$ or $PR = -6$ (rej.)

$PQ = 6 \times 2 = 12$ units

9.

$\because P$ is equidistant from BC

$\therefore MN // BC$

$\Delta AMN \sim \Delta ABC$

$$\left(\frac{AM}{AB}\right)^2 = \frac{\text{Area of } \Delta AMN}{\text{Area of } \Delta ABC}$$

$$\left(\frac{AM}{AB}\right)^2 = \frac{1}{4}$$

$$\frac{AM}{AB} = \frac{1}{2}$$

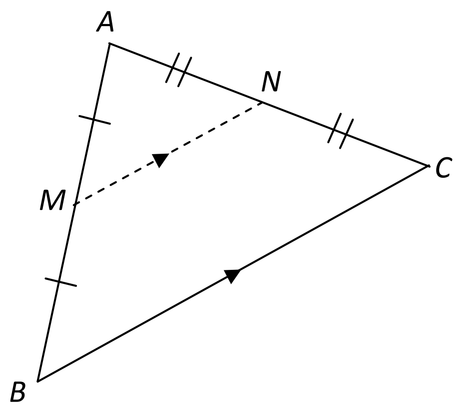
M is the mid-point of AB.

$$\left(\frac{AN}{AC}\right)^2 = \frac{\text{Area of } \Delta AMN}{\text{Area of } \Delta ABC}$$

$$\left(\frac{AN}{AC}\right)^2 = \frac{1}{4}$$

$$\frac{AN}{AC} = \frac{1}{2}$$

N is the mid-point of AC.



10. (a)

The number of arrangements

$= 9! = 362880$

(b)

The number of arrangements

$= 6!4! = 17280$

(c)

The number of arrangements

$= 5!4! = 2880$

11. (a)

The number of ways

$= C_1^1 C_5^{14} = 2002$

(b)

The number of ways

$= \left(C_3^{10} C_3^5 + C_4^{10} C_2^5 \right) \times 6! = 2376000$

12.

Let x km/hr be the speed of Express Rail.

The speed of Dick Hui $= (x+115)$ km/hr

$$\frac{1150}{x} - \frac{1150}{x+115} = \frac{1}{2}$$

$$1150(x+115) - 1150x = \frac{1}{2}x(x+115)$$

$$x^2 + 115x - 264500 = 0$$

$$(x+575)(x-460) = 0$$

$$x = 460 \text{ or } x = -575 \ (\text{rej.})$$

The speed of Express Rail $= 460$ km/hr

The speed of Dick Hui $= 460 + 115 = 575$ km/hr

13. (a)

Sub $y=0$,

$x^2-6x+5=0$

$(x-5)(x-1)=0$

$x=5$ or $x=1$

The coordinates of A is $(5,\ 0)$.

The coordinates of B is $(1,\ 0)$.

(b) (i)

P and K is solution of $\begin{cases} y=x^2-6x+5 \\ y=2x+1 \end{cases}$

$x^2-6x+5=2x+1$

$x^2-8x+4=0$

The sum of roots $= p+k=-\frac{-8}{1}=8$

The product of roots $= pk=\frac{4}{1}=4$

$\frac{1}{p^2}+\frac{1}{k^2}=\frac{k^2+p^2}{p^2k^2}=\frac{(p+k)^2-2pk}{(pk)^2}=\frac{8^2-2(4)}{4^2}=\frac{56}{16}=\frac{7}{2}$

$p^2+8k=p^2+(p+k)k=p^2+pk+k^2=(p+k)^2-pk=8^2-4=60$

Since p is the roots of $x^2-8x+4=0$, $p^2-8p+4=0$, $p^2-8p=-4$

$p^2-8p+1=-4+1=-3$

(ii)

From (b)(i), $p+k=8$

The x-coordinate of mid-point of $PK=\frac{p+k}{2}=4$

The mid-point of PK is also on the line $y=2x+1$.

$\therefore$ Put $x=4$ into $y=2x+1$,

$y=2(4)+1$

$y=9$

The coordinates of mid-point of PK is (4, 9).

14. (a)

$\angle AOC = 2\times\angle ABC$ ($\angle$ at centre $= 2$ $\angle$ at circumference)

$= 2\times 30°$

$= 60°$

$OA = OC$ (radii in the same circle)

$\angle OAC = \angle OCA$ (base $\angle$s, isos. Δ)

$\angle OAC + \angle OCA + 60° = 180°$ ($\angle$ sum of Δ)

$\angle OAC = 60°$

$CA = CD$ (given)

$\angle CAD = \angle CDA$ (base $\angle$s, isos. Δ)

$\angle AOC + \angle OAC + \angle CAD + \angle CDA = 180°$ ($\angle$ sum of Δ)

$\angle CAD = 30°$

$\angle OAD = \angle OAC + \angle CAD$

$= 60° + 30°$

$= 90°$

$\because \angle OAD = 90°$

$OA \perp AD$

$\therefore AD$ is the tangent of the circle.

(b)

BC

$= \sqrt{(6-2)^2 + (2-5)^2}$

$= 5$ units

Let E be a point where OD intersects with AB.

$\sin 30° = \frac{CE}{BC}$

$CE = 5\sin 30°$

$AE \perp OC$

$OE = CE$ (property of isos. Δ)

$OE = 5\sin 30°$ units

$OC = 10\sin 30°$

$OA = OC$ (radii in the same circle)

$OA = 10\sin 30°$

$\tan 30° = \frac{OA}{AD}$

$AD = \frac{10\sin 30°}{\tan 30°}$

$AD = 5\sqrt{3}$ units

15. (a)

Let $(x,\ y)$ be the coordinates of P.

$$AP = BP$$

$$\sqrt{(x-7)^2+\left[y-(-1)\right]^2}=\sqrt{(x-3)^2+(y-9)^2}$$

$$(x-7)^2+(y+1)^2=(x-3)^2+(y-9)^2$$

$$\left(x^2-14x+49\right)+\left(y^2+2y+1\right)=\left(x^2-6x+9\right)+\left(y^2-18y+81\right)$$

$$8x-20y+40=0$$

$$2x-5y+10=0$$

The required equation is $2x-5y+10=0$.

(b)

Let $(x,\ y)$ be the coordinates of P.

$\because AP \perp BP$

$$\therefore \quad \frac{y-(-1)}{x-7}\times\frac{y-9}{x-3}=-1$$

$$\frac{y+1}{x-7}\times\frac{y-9}{x-3}=-1$$

$$\frac{y^2-8y-9}{x^2-10x+21}=-1$$

$$y^2-8y-9=-x^2+10x-21$$

$$x^2+y^2-10x-8y+12=0$$

The required equation is $x^2+y^2-10x-8y+12=0$.

Paper I (Set 2) Suggested solution

1. (a)

Sub $y=0$,

$$x^2+0^2+4x-4(0)-12=0$$

$$x^2+4x-12=0$$

$$(x+6)(x-2)=0$$

$$x=-6 \text{ or } x=2$$

$\because a>b$, $\therefore a=2, b=-6$

The coordinates of A and B are $(2,\ 0)$ and $(-6,\ 0)$.

(b)

The coordinates of the centre $=\left(-\frac{4}{2},\ -\frac{-4}{2}\right)=(-2,\ 2)$

Let the coordinates of P be $(a,\ b)$.

$\frac{a+2}{2}=-2,\ \frac{b+0}{2}=2$

$\therefore a=-6,\ b=4$

The coordinates of $P=(-6,\ 4)$

$\therefore BP=4-0=4$ units

2. (a)

The centre of $C_1 = \left(-\frac{2}{2},\ -\frac{-4}{2}\right) = (-1,\ 2)$

The centre of $C_2 = \left(-\frac{2}{2},\ -\frac{-4}{2}\right) = (-1,\ 2)$

C_1 and C_2 are concentric circles.

Let O be their centres.

Let D be a point on AB where C_1 touches AB at D.

$OD \perp AB$ (tangent $\perp$ radius)

$\because OD \perp AB$

$\therefore AD = BD$ (line from centre $\perp$ chord bisects chord)

The radius of $C_1 = OD = \sqrt{1^2 + 2^2 - (-4)} = 3$

The radius of $C_2 = OB = \sqrt{1^2 + 2^2 - (-20)} = 5$

$OD^2 + BD^2 = OB^2$ (Pyth. Theorem)

$3^2 + BD^2 = 5^2$

$BD = 4$ or $BD = -4$(rej.)

$AB = 4 \times 2 = 8$ units

(b)

$\because M_{OD} = 0$

$\therefore M_{AB}$ is infinite.

AB is a vertical line. Its equation is $x = 2$.

3.

P(all different)

$= 1 \times \frac{5}{6} \times \frac{4}{6} \times \frac{3}{6}$

$= \frac{5}{18}$

4.

The probability of Wai Ming passes the test

= 3 questions answer correctly OR 4 questions answer correctly

$= 4(\frac{1}{5}\times\frac{1}{5}\times\frac{1}{5}\times\frac{4}{5}) + \frac{1}{5}\times\frac{1}{5}\times\frac{1}{5}\times\frac{1}{5}$

$= \frac{17}{625}$

5. (a)

$N = kt^2$ where $k \neq 0$

Sub $N = 10000$ and $t = 4$,

$10000 = k(4)^2$

$k = 625$

$N = 625t^2$

Sub $t = 8$,

$N = 625(8)^2$

$= 40000$

The number of students is 40000.

(b)

Let N be the number of hair of Mr. Chau.

$N = \frac{k}{t^2}$ where $k \neq 0$

Sub $N = 100$ and $t = 4$,

$100 = \frac{k}{4^2}$

$k = 1600$

$N = \frac{1600}{t^2}$

Sub $t = 8$,

$N = \frac{1600}{8^2}$

$= 25$

The number of hair is 25.

6. (a)

The median

$= \text{the } 4.5^{th} \text{ term}$

$= \frac{1}{2}(60 + 65)$

$= 62.5$

The inter-quartile range

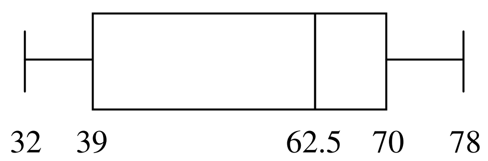
$= \text{the } 7^{th} \text{ term} - \text{the } 2^{nd} \text{ term}$

$= 70 - 39$

$= 31$

(b)

<u>Mathematics exam results of a group of students</u>



(c)

The new median

$= 62.5 + 20$

$= 82.5$

The inter-quartile range

$= 31$

7. (a)

$$43-(10+y)=29$$
$$y=4$$

$$(30+x)-19=17$$
$$x=6$$

(b) (i)

The median will change.
Original: Median is 30.
New: Median is 26.

(ii)

The mode will change.
Original: Mode is 30.
New: Mode is 24 and 30.

(iii)

The range will remain unchanged.

(iv)

The inter-quartile range will change.
Original: Inter-quartile range is 17.
New: Inter-quartile range is $34-19=15$.

8.

$$\frac{-2+0+4+a+b}{5}=1$$

$a+b=3$ ---(1)

$$\sqrt{\frac{(-2-1)^2+(0-1)^2+(4-1)^2+(a-1)^2+(b-1)^2}{5}}=2$$

$a^2-2a+b^2-2b+1=0$ ---(2)

From (1), $a=3-b$ ---(3)

Sub $a=3-b$ into (2),

$(3-b)^2-2(3-b)+b^2-2b+1=0$

$2b^2-6b+4=0$

$b^2-3b+2=0$

$(b-1)(b-2)=0$

$b=1$ or 2

Sub $b=1$,

$a=3-1$

$=2$

Sub $b=2$,

$a=3-2$

$=1$

9.

$80°+\angle ABC=180°$ (int. $\angle$ of // lines)

$\angle ABC=100°$

$$\frac{24}{\sin\angle ABC}=\frac{AB}{\sin 50°}$$

$$\frac{24}{\sin 100°}=\frac{AB}{\sin 50°}$$

$AB=18.7$ (corr. to 3 sig. fig.)

The distance between Faye Faye and Dick Hui is 18.7 km now.

10. (a)

The distance

$= \sqrt{6^2 + 9^2 - 2(6)(9)\cos 120°}$

$= \sqrt{171}$ km

(b)

$$\frac{6}{\sin(\theta - 20°)} = \frac{\sqrt{171}}{\sin 120°}$$

$\theta = 43.4°$ (corr. to 3 sig. fig.)

The bearing of Faye Faye from Dick Hui is $S43.4°E$.

11.

Add a point E on AB so that $DE//CB$.

$\angle DEA = 75°$ (corr. $\angle$s, $DE//CB$)

$\angle ADE = 180° - 62° - 75° = 43°$ ($\angle$ sum of Δ)

In ΔADE,

$$\frac{DE}{\sin 62°} = \frac{AE}{\sin 43°}$$

$$AE = \frac{8\sin 43°}{\sin 62°} \approx 6.17929$$

$BE = 12 - 6.17929 = 5.82071 \approx 5.82$

$CD = BE = 5.82$ cm

12. (a)

AB

$= \sqrt{8^2 + 10^2 - 2(8)(10)\cos 40°}$

$= 6.4368$

$= 6.44$ cm (corr. to 3 sig. fig.)

(b)

$$\frac{1}{2}(8)(10)\sin 40° = \frac{1}{2}(6.4368)(CD)$$

$CD = 7.9889$

$CD \approx 7.99$ cm (corr. to 3 sig. fig.)

(c)

Let θ be the angle between the line *VD* and the plane *ABC*.

$$\tan\theta = \frac{6}{CD}$$

$$\tan\theta = \frac{6}{7.9889}$$

$$\theta = 36.9° \text{ (corr. to 3 sig. fig.)}$$

13.

Let *a* be the 1st term.

Let *r* be the common ratio.

The 2nd term $= ar$

The 4th term $= ar^3$

The 5th term $= ar^4$

$$a + ar = 216$$

$$a(1+r) = 216 \quad \text{---(1)}$$

$$ar^3 + ar^4 = -8$$

$$ar^3(1+r) = -8 \text{ ---(2)}$$

$(2) \div (1)$:

$$r^3 = -\frac{8}{216}$$

$$r = -\frac{1}{3}$$

Sub $r = -\frac{1}{3}$ into (1):

$$a\left(1 - \frac{1}{3}\right) = 216$$

$$a = 324$$

The 1st term is 324.