Paper I (Set 1) Suggested solution

1.

$$\alpha+\beta=-\frac{-3}{2}=\frac{3}{2}$$

$$\alpha\beta=\frac{-4}{2}=-2$$

$$(a)\ \left(\frac{\alpha^2}{1-\beta}\right)\left(\frac{\beta^2}{1-\alpha}\right)=\frac{\alpha^2\beta^2}{(1-\beta)(1-\alpha)}=\frac{(\alpha\beta)^2}{1-(\alpha+\beta)+\alpha\beta}=\frac{(-2)^2}{1-(\frac{3}{2})+(-2)}=-\frac{8}{5}$$

$$(b)\ \alpha-\beta=\sqrt{(\alpha+\beta)^2-4\alpha\beta}=\sqrt{(\frac{3}{2})^2-4(-2)}=\sqrt{\frac{41}{4}}=\frac{\sqrt{41}}{2}$$

$$(c)\ 2\alpha^2-3\alpha=2\alpha^2-2(\alpha+\beta)\alpha=2\alpha^2-2\alpha^2-2\alpha\beta=-2\alpha\beta=-2(-2)=4$$

2.

Put $x=0$, $y=0^2+3(0)+18=18,$ so C=(0,18)

. Put $y=0,\ 0=-x^2+3x+18$ => $x=-3$ or $x=6$

So A=(-3,0),B=(6,0)

3.

$(a)\ f(-1)=(-1)^2-4(-1)+8=13$

$(b)\ f(x+1)-f(x)=[(x+1)^2-4(x+1)+8]-[x^2-4x+8]$

$=x^2+2x+1-4x-4+8-x^2+4x-8=2x-3$

$(c)\ f(k)=13,\ k^2-4k+8=13,\ k^2-4k-5=0,\ (k-5)(k+1)=0,\ k=5$ or -1

4.

(a) $3x^2+4x+k=0$ has real roots => $\Delta\geq 0$

$4^2-4(3)(k)\geq 0$

16-12k ≥ 0

k $\leq \frac{4}{3}$

(b) As k is a positive integer , $k=1$ only.

5.

Method 1:

$x+2\sqrt{x\text{-}1}-1=0$

$(x-1)+2\sqrt{x-1}=0$

Let $y=\sqrt{x\text{-}1}$, then

. $y^2+2y=0$

$y(y+2)=0$

$y=0$ or $y=\text{-}2$

$\sqrt{x\text{-}1}=0$ or $\sqrt{x\text{-}1}=-2(rejected)$

So $x=1$.

Method 2:

$x+2\sqrt{x\text{-}1}-1=0$

$2\sqrt{x-1}=1-x$

$4(x-1)=(1-x)^2$

$4-4x=1-2x+x^2$

$x^2-6x+5=0$

$(x\text{-}1)(x\text{-}5)=0$

$x=1$ or $x=5(rejected)$

6.

$f(x)=g(x)+(f(x))^2+1$

$f(x)=-f(x)+(f(x))^2+1$

$(f(x))^2-2f(x)+1=0$

$(f(x)-1)^2=0$

$f(x)=1$

7.

$2x^4-5x^2-12=0$

Let $y=x^2$, then

$2\text{y}^2-5y-12=0$

$(2y+3)(y-4)=0$

$y=4$ or $y=-\dfrac{3}{2}$ (rejected)

$\text{x}^2=4$

$\text{x}=\pm 2$

8.

$$\alpha+\beta=-\frac{k+2}{2},\ \alpha\beta=\frac{-2}{2}=-1$$

$$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=-\frac{17}{4}\Rightarrow\frac{\alpha^2+\beta^2}{\alpha\beta}=-\frac{17}{4}\Rightarrow\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}=-\frac{17}{4}$$

$$\frac{(-\frac{k+2}{2})^2-2(-1)}{-1}=-\frac{17}{4}\Rightarrow\frac{k^2+4k+4}{4}+2=\frac{17}{4}$$

$$k^2+4k+4+8=17$$

$$k^2+4k-5=0$$

$$(k-1)(k+5)=0$$

$k=1$ or -5

9.

$$x^2-(a+1)x+a^2-6a+5=0$$

Let the two roots be $\alpha,\ 3\alpha$

$$\alpha+3\alpha=-\frac{-(a+1)}{1}\Rightarrow 4\alpha=a+1\text{.................................(1)}$$

$$\alpha(3\alpha)=\frac{a^2-6a+5}{1}\Rightarrow 3\alpha^2=a^2-6a+5\text{....................(2)}$$

From (1), $\alpha=\frac{(a+1)}{4}$............(3)

Put (3) into (2),

$$3\left[\frac{(a+1)}{4}\right]^2=a^2-6a+5$$

$$\frac{3(a^2+2a+1)}{16}=a^2-6a+5$$

$$3a^2+6a+3=16a^2-96a+80$$

$$13a^2-102a+77=0$$

$$(13a-11)(a-7)=0$$

$a=\frac{11}{13}$ or 7

10. (a)

Let the coordinates of P be (x, y).

$$\frac{2(x)+3(4)}{2+3}=6 \Rightarrow x=9$$

$$\frac{2(y)+3(5)}{2+3}=9 \Rightarrow y=15$$

The coordinates of P is (9, 15).

(b)

Distance of $AP=\sqrt{(9-4)^2+(15-5)^2}=\sqrt{125}=5\sqrt{5}$

11. (a)

Let the mid-point of AB be M.

The coordinate of $M=(\frac{-3+9}{2},\ \frac{4+8}{2})=(3,\ 6)$

Slope of $AB=\frac{8-4}{9-(-3)}=\frac{1}{3}$

Slope of $M=-1\div\frac{1}{3}=-3$

$\therefore$ Equation of L: $\frac{y-6}{x-3}=-3 \Rightarrow 3x+y-15=0$

(b)

Put $y=0$ into $3x+y-15=0$,

$3x+0-15=0$

$x=5$

$\therefore$ The coordinates of C is (5, 0).

12.

(a) Axis of symmetry : $x=5$

(b) As (h,k) in the equation $y=a(x-h)^2+k$ represents the vertex.

Clearly h=5 by (a)

Put (3,10) and (6,16):

$$\begin{cases} 10 = a(3-5)^2 + k \\ 16 = a(6-5)^2 + k \end{cases}$$

. On solving , we get $a = -2, k = 18$

So the equation is : $y = -2(x-5)^2 + 18$

(c) Maximum value of y is 18.

13.

(a) $h = -2t^2 + 8t + 1$

$h = -2(t^2 - 4t + 2^2) + 1 + 2(2^2)$

$h = -2(t-2)^2 + 9$

so at $t = 2$ seconds, he will reach the maximum height

(b) By (a), his maximum height is 9 , so Faye Faye will not collide with it.

(c) When t=1, h=7.

When t=4, h=1.

So at the time interval $1 \le t \le 4$, the minimum height is 1.

14.

(a) Δ=49 => $k^2 - 4(-1)(10) = 49$

$k^2 = 9$

$k = 3$, or -3(rejected)

(b) Put $y = 0$, $-x^2$+3x+10=0

$x^2 - 3x - 10 = 0$

$(x-5)(x+2) = 0$

$x = 5$ or $x = -2$

so A=(-2,0), B=(5,0)

$$y = -x^2 + 3x + 10$$

$$= -(x^2 - 3x + \left(\frac{3}{2}\right)^2) + 10 + \left(\frac{3}{2}\right)^2$$

$$= -(x - \frac{3}{2})^2 + \frac{49}{4}$$

As C is the vertex, so C=($\frac{3}{2}$,$\frac{49}{4}$)

(c) When $y = 6$, we have $6 = -x^2 + 3x + 10$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

so $x = 4$ *or* $x = -1$

Hence E=(-1,6), F=(4,6)

Area of ΔCEF:$\frac{1}{2}$(1+4)($\frac{49}{4}$-6)=$\frac{125}{8}$

Area of trapezium: $\frac{1}{2}(4+5)(6) = 27$

Area of OECFD:$\frac{125}{8}$+27=42.625 (sq.units)

Paper I (Set 2) Suggested solution

1. (a) $p^2 - 36q^2 = p^2 - (6q)^2$

$$= (p + 6q)(p - 6q)$$

(b) $p^2 - 36q^2 + 5p + 30q = (p + 6q)(p - 6q) + 5p + 30q$ By (a)

$$= (p + 6q)(p - 6q) + 5(p + 6q)$$

$$= (p + 6q)(p - 6q + 5)$$

2. $x = \angle ABD = 40^\circ$ (base$\angle$, isosΔ)

$z = x = 40^\circ$ (alt $\angle$s, AE//BD)

$\angle CAD = x = 40^\circ$ (base$\angle$, isosΔ)

$\angle BAE + \angle ABD = 180^\circ$ (int $\angle$s, AE//BD)

$(40^\circ + 40^\circ + y) + 40^\circ = 180^\circ$

$$y = 60^\circ$$

3. (a) $\angle EAF = \angle DAC$ (Common)

$\angle AEF = \angle ACD$ (corr $\angle$s, DC//EB)

$\angle AFE = \angle ACD$ ($\angle$ sum of Δ)

$\therefore \Delta AEF \sim \Delta ADC$ (A.A.A.)

$\angle EAF = \angle CBF$ (Given)

$\angle AFE = \angle BFC$ (Vert. oppo. $\angle$)

$\angle FEA = \angle FCB$ ($\angle$ sum of Δ)

$\therefore \Delta AEF \sim \Delta BCF$ (A.A.A.)

$\therefore \Delta AEF \sim \Delta ADC \sim \Delta BCF$ (A.A.A.)

(b) $\because \Delta AEF \sim \Delta ADC \sim \Delta BCF$ (Proved)

$\frac{AE}{AD} = \frac{AF}{AC}$ (corr. sides, $\sim \Delta$)

$\frac{AE}{AE+3} = \frac{6}{6+4}$

$5AE = 3AE + 9$

$AE = 4.5$

$\frac{AE}{BC} = \frac{EF}{CF}$ (corr. sides, $\sim \Delta$)

$\frac{4.5}{6} = \frac{EF}{4}$

$EF = 3$

$\frac{AF}{BF} = \frac{EF}{CF}$ (corr. sides, $\sim \Delta$)

$\frac{6}{BF} = \frac{3}{4}$

$BF = 8$

$\therefore BE = EF + BF = 3 + 8 = 11$

4. $BC^2 = 7^2 + 11^2$

$BC = \sqrt{170}$

$BK = AK = \frac{BC}{2} = \frac{\sqrt{170}}{2}$

By cosine formula

$$\cos\theta = \frac{AK^2 + BK^2 - AB^2}{2(AK)(BK)}$$

$$\cos\theta = \frac{\frac{170}{4} + \frac{170}{4} - 7^2}{2\left(\frac{170}{4}\right)}$$

$$\cos\theta = \frac{36}{85}$$

$\theta = 64.9^{\circ}$ (1 d. p.)

5. (a) $f(-k) = -k$

$(2-k)(-k-k)+k = -k$

$-4k + 2k^2 + k = -k$

$2k^2 - 2k = 0$

$k(k-1) = 0$

$k = 0 \quad \text{or} \quad k = 1$

(b) $k = 0$

$f(x) = x(x+2)$

$f(x) = x^2 + 2x$

$\therefore$ **The constant term is 0 and the coefficient of x is 2.**

(c) $k = 2$

$f(x) = (x+2)(x-2)+2$

$f(x) = x^2 - 2$

$x^2 - 2 = 0$

$x^2 = 2$

$x = \pm\sqrt{2}$

6. (a) $2x^2 + x - 2 = 0$

$$x = \frac{-1 \pm \sqrt{1 - 4(2)(-2)}}{2(2)} = \frac{-1 \pm \sqrt{17}}{4}$$

(b) $2\cos^2\theta - \sin\theta = 0$

$2\left(1 - \sin^2\theta\right) - \sin\theta = 0$

$2 - 2\sin^2\theta - \sin\theta = 0$

$2\sin^2\theta + \sin\theta - 2 = 0$

$\sin\theta = \dfrac{-1 \pm \sqrt{17}}{4}$ (by (a))

$\theta = 51.3^\circ$ or $\theta = 128.7^\circ$

7. (a) $f(-4) = 0$

$3(-4)^3 + 14(-4)^2 + k = 0$

$k = -32$

(b) $f(x) = 3x^3 + 14x^2 - 32$

$f(x) = (x + 4)\left(3x^2 + 2x - 8\right)$

$f(x) = (x + 4)(3x - 4)(x + 2)$

8. Sub (0, 2) into $y = A + B\sin x^\circ$

$2 = A + B\sin 0^\circ \Rightarrow 2 = A$

$\therefore y = 2 + B\sin x^\circ$

Sub (90, 5) into $y = 2 + B\sin x^\circ$

$5 = 2 + B\sin 90^\circ$

$B = 3$

9. (a) $\log 14 = \log(2 \times 7) = \log 2 + \log 7 = a + c$

(b) $\log 24 = \log(2^3 \times 3) = \log 2^3 + \log 3 = 3\log 2 + \log 3 = 3a + b$

(c) $\log \dfrac{49}{12} = \log 7^2 - \log(2^2 \times 3) = 2\log 7 - 2\log 2 - \log 3 = 2c - 2a - b$

10. $\dfrac{(a^{-2}b)^2}{a^4} = \dfrac{a^{-4}b^2}{a^4} = a^{-8}b^2 = \dfrac{b^2}{a^8}$

11. $\begin{cases} 3^{x+y} = 27 = 3^3 \\ 3^{x-2y} = 1 = 3^0 \end{cases}$

$\Rightarrow \begin{cases} x + y = 3 - - - (1) \\ x - 2y = 0 - - - (2) \end{cases}$

On solving (1) and (2) , $x = 2,\ y = 1$