Paper II (Set 1) Suggested solution

1.B	6.C	11.C	16.D	21.A	26.C	31.B	36.C
2.C	7.D	12.A	17.C	22.C	27.D	32.A	37.B
3.C	8.D	13.B	18.A	23.A	28.C	33.D	38.A
4.C	9.B	14.C	19.A	24.B	29.C	34.B	39.A
5.A	10.C	15.D	20.A	25.C	30.C	35.B	40.B

1. The answer is B

$x-1=\sqrt{(5-5x)}$

$(x-1)^2=5-5x$

$x^2-2x+1=5-5x$

$x^2+3x-4=0$

$(x-1)(x+4)=0$

$x=1$ or -4 (Rejected)

2. The answer is C

$2x(x+5)=5(x+5)$

$(2x-5)(x+5)=0$

$x=\frac{5}{2}$ or $x=$ -5

3. The answer is C

If $x^2-2x+k=1$ has no real roots , then $\Delta<0$.

$(-2)^2-4(1)(k-1)<0$

$1-(k-1)<0$

$k>2$

4. The answer is C

If $5x^2 - ax + 20 = 0$has equal positive roots , then $\Delta = 0$.

$(a)^2 - 4(5)(20) = 0$

$a = \pm 20$

When $a = -20$, $5x^2 + 20x + 20 = 0$ => $x = -20$

When $a = 20$, $5x^2 - 20x + 20 = 0$ => $x = 20$

As the root is positive , so $a = 20$

5. The answer is A

$\sqrt{2 - x} = x$

$\left(\sqrt{2 - x}\right)^2 = x^2$

$2 - x = x^2$

$x^2 + x - 2 = 0$

$x = 1$ or $x = -2$ (rejected)

6. The answer is C

Let $y = 2x + 1$

$y^2 - 3y + 2 = 0$

$y = 1$ or $y = 2$

$2x + 1 = 1$ or $2x + 1 = 2$

$x = 0$ or $x = \frac{1}{2}$

7. The answer is D

$3x^2 - 6x - 21 = 0$

$x^2 - 2x - 7 = 0$

$x = \frac{2 \pm \sqrt{2^2 - 4(-7)}}{2} = 1 \pm 2\sqrt{2}$

8. The answer is D

I. (True)

$2x^2 - x + 1 = 0$

$\Delta = (-1)^2 - 4(2)(1) = -7 < 0$ ==> No real roots

II. (True)

$x^2 + ax - 1 = 0$

$\Delta = (a)^2 - 4(1)(-1) = a^2 + 4 > 0$ ==> 2 distinct real roots

III. (True)

$y = x^2 + 2x + 1 \; = (x+1)^2 = (x-(-1))^2 + 0$

So the minimum value of y is 0

9. The answer is B

$a > 0$ ==> open upwards

$\Delta > 0$ ==> 2 roots

$a > 0, b < 0$ ==> tends to the right. $(-\frac{b}{2a}<0)$

10. The answer is C

$y = x^2 - 6x - 7$

$y = (x-3)^2 - 16$

So the vertex is $(3, -16)$

11. The answer is C

$a < 0$ ==> open downwards

$c > 0$ ==> y-intercept > 0

$a > 0, \; b = 0$ ==> tends to the middle.

12. The answer is A

open upwards

$\therefore a > 0$

tends to the left

$\therefore b > 0$ (as $a > 0$)

y-intercept < 0

$\therefore c < 0$

13. The answer is B

As the vertex is (4, 8),

So $y = a(x-4)^2 + 8$

Put (6, 0),

$0 = 4a + 8$ ==> $a = -2$

14. The answer is C

Axis of symmetry: $x = \dfrac{-8+0}{2} \Rightarrow x = -4$

15. The answer is D

$\Delta = 0$

$(-8)^2 - 4c = 0$

$c = 16$

16. The answer is D

open downwards ==> $a < 0$

y-intercept < 0 ==> $c < 0$

No real root ==> $\Delta < 0$

17. The answer is C

Let x and y be the lenght and width of the rectangle.

Perimeter: $2(x + y) = 52$

=> $x = 26 - y$

Area: $xy = 153$

$(26 - y)y = 153$

$y^2 - 26y + 153 = 0$

$y = 9$ or $\mathrm{y} = 17$

=> $x = 17$ or $x = 9$

So the dimension is 9×17

18. The answer is A

$$\alpha+\beta=-\frac{3}{2},\ \alpha\beta=-\frac{1}{2}$$

$$\alpha+\frac{\beta^2}{\alpha+\beta}=\frac{\alpha(\alpha+\beta)+\beta^2}{\alpha+\beta}=\frac{\alpha^2+\alpha\beta+\beta^2}{\alpha+\beta}=\frac{(\alpha+\beta)^2-\alpha\beta}{\alpha+\beta}$$

$$=\frac{(-\frac{3}{2})^2-(-\frac{1}{2})}{-\frac{3}{2}}=-\frac{11}{6}$$

19. The answer is A

$a+b=3,\ ab=-1$

$(a^2-b)+(b^2-a)=a^2+b^2-a-b=(a+b)^2-2ab-(a+b)$

$=(3)^2-2(-1)-3=8$

$(a^2-b)(b^2-a)=(ab)^2-a^3-b^3+ab=(ab)^2+ab-(a^3+b^3)$

$=(ab)^2+ab-(a+b)(a^2-ab+b^2)=(ab)^2+ab-(a+b)[(a+b)^2-3ab]$

$=(-1)^2-1-(3)[(3)^2-3(-1)]=-36$

$\therefore$ The equation with roots a^2-b and b^2-a is $x^2-8x-36=0$

20. The answer is A

Since $\frac{\alpha^2}{k}-2\alpha+p=0$ and $\frac{\beta^2}{k}-2\beta+p=0$, α and β are the roots of the equation $\frac{x^2}{k}-2x+p=0$.

$$\alpha\beta=\frac{p}{\frac{1}{k}}=pk$$

21. The answer is A

Axis of symmetry : $x-3=0\Rightarrow x=3$

22. The answer is C

$\alpha+\beta=5,\ \alpha\beta=2$

$\frac{3\alpha^2}{5}+3\beta=\frac{3\alpha^2}{5}+\frac{3(\alpha+\beta)}{5}\beta=\frac{3\alpha^2}{5}+\frac{3\alpha\beta+3\beta^2}{5}=\frac{3}{5}(\alpha^2+\alpha\beta+\beta^2)$

$=\frac{3}{5}[(\alpha+\beta)^2-\alpha\beta]=\frac{3}{5}[(5)^2-2]=\frac{69}{5}$

23. The answer is A

$f(x)f(x-1)=x(x-1)(x-1)(x-2)=x(x-1)^2(x-2)$

24. The answer is B

$f(2)=1$

=> $2f(2)+b=1$

=> $2(1)+b=1$

=> $b=-1$

25. The answer is C

$f(-1)=f(1)$

=> $a-b+c\ \ =a+b+c$

=> $b=0$

26. The answer is C

$f(x+1)=x^2+3x+1$

$f((x-1)+1)=(x-1)^2+3(x-1)+1=\ x^2+x-1$

$f(x)=\ x^2+x-1$

27. The answer is D

I.(false): counter example: $f(x)=x,\ f(-x)=-x$, so $f(x)\neq f(-x)$

II.(false): counter example: $f(x)=x^2,\ f(x+1)=(x+1)^2$, so $f(x)+1\neq f(x+1)$

III. (false): counter example: $f(x)=x,\ f(x)+f(\frac{1}{x})=x+\frac{1}{x}\neq 1=f(1)$

28. The answer is C

I.(true): $x^2 = (-x)^2 \Rightarrow f(x) = f(-x)$

II.(false)

III.(true): $(x-2)^2 = (2-x)^2 \Rightarrow f(x-2) = f(2-x)$

29. The answer is C

$f(x) = 2x+1$

$f(f(x)) = f(2x+1) = 2(2x+1)+1 = 4x+3$

$f(f(f(x))) = f(4x+3) = 2(4x+3)+1 = 8x+7$

30. The answer is C

$f(1) - f(2) + f(3) - f(4) + ... + f(9) - f(10)$

$= 1-0+1-0+1-0+1-0+1-0$

$= 5$

31. The answer is B

The domain of $f(x)$ is all real numbers except $\sqrt{x-2} - 2 \neq 0,\ x-2 \geq 0$

$\therefore$ The domain of $f(x)$ is all real numbers greater than 2 except 6.

32. The answer is A

$f(x) = 2x+1,\ g(x) = x+1$

$f(g(x)) = f(x+1) = 2(x+1)+1 = 2x+3$

33. The answer is D

34. The answer is B

$f(x) = x^2 - 3x + 10 = \left(x - \frac{3}{2}\right)^2 + \frac{31}{4}$

so the minimum value of $f(x)$ is $\frac{31}{4}$

35. The answer is B

Let Keith's time be *x*.

Rate of Zhu $= \frac{1}{x}$ Rate of Keith $= \frac{1}{x-10}$

$$\frac{1}{x} + \frac{1}{x-10} = \frac{1}{x-18}$$

$$\frac{x-10+x}{x^2-10x} = \frac{1}{x-18}$$

$$2x^2-36x-10x+180 = x^2-10x$$

$$x^2-36x+180=0$$

$$(x-30)(x-6)=0$$

$$x=6\ (\textit{rejected})\text{ or }30$$

36. The answer is C

The straight line passes through the point $(-9,\ 0)$ and $(0,\ 2)$.

The equation of straight line:

$$\frac{y-2}{x-0} = \frac{2-0}{0-(-9)}$$

$$\frac{y-2}{x} = \frac{2}{9}$$

$$9y-18=2x$$

$$2x-9y+18=0$$

37. The answer is B

Method 1:

$$OM = \sqrt{(4-0)^2+(-3-0)^2} = 5$$

By mid point theorem, $AC = 2\times OM = 10$

Method 2:

Let the coordinate of A be $(a,\ b)$ and the coordinate of C be $(c,\ d)$.

$$\frac{a+1}{2}=0,\ \frac{b+(-4)}{2}=0,\ \frac{c+1}{2}=4,\ \frac{d+(-4)}{2}=-3$$

$$a=-1,\ b=4,\ c=7,\ b=-2$$

$$AC = \sqrt{(-1-7)^2+(4-(-2))^2} = 10$$

38. The answer is A

Slope of $2x-3y+9=0: -\frac{2}{-3}=\frac{2}{3}$

Slope of $6x+4y-5=0: -\frac{6}{4}=-\frac{3}{2}$

Since $\frac{2}{3}\times(-\frac{3}{2})=-1$, they are perpendicular to each other.

39. The answer is A

Let the coordinate of C be $(x,\ y)$.

Slope of $BC=$ Slope of AD

$\frac{y-5}{x-4}=\frac{2-(-2)}{-5-(-7)}\Rightarrow\frac{y-5}{x-4}=2\Rightarrow 2x-y-3=0$..................(1)

Slope of $DC=$ Slope of AB

$\frac{y-(-2)}{x-(-7)}=\frac{5-2}{4-(-5)}\Rightarrow\frac{y+2}{x+7}=\frac{1}{3}\Rightarrow x-3y+1=0$................(2)

By solving (1) and (2),

$x=2,\ y=1$

Slope of $AC=\frac{1-2}{2-(-5)}=-\frac{1}{7}$

40. The answer is B

Slope of $kx+5y-8=0: -\frac{k}{5}$

Slope of $2x-7y+5=0: -\frac{2}{-7}=\frac{2}{7}$

$-\frac{k}{5}=\frac{2}{7}$

$k=-\frac{10}{7}$

Paper II (Set 2) Suggested solution

1.A	6.B	11.B	16.D	21.D	26.B	31.C	36.B
2.C	7.D	12.A	17.A	22.D	27.D	32.D	37.C
3.D	8.C	13.A	18.A	23.D	28.C	33.A	38.C
4.D	9.B	14.C	19.B	24.C	29.C	34.D	39.C
5.D	10.C	15.C	20.A	25.C	30.D	35.B	40.B

1. The answer is A

$$\begin{aligned}\frac{\sqrt{3}\tan 30^\circ \cos\theta}{2\tan(90^\circ-\theta)\sin 30^\circ} &= \frac{\sqrt{3}\left(\frac{1}{\sqrt{3}}\right)\cos\theta}{2\left(\frac{1}{\tan\theta}\right)\left(\frac{1}{2}\right)} \\ &= \cos\theta\tan\theta \\ &= \cos\theta\left(\frac{\sin\theta}{\cos\theta}\right) \\ &= \sin\theta\end{aligned}$$

2. The answer is C

$$\sin(180^\circ+\theta)+\cos(270^\circ+\theta) = -\sin\theta+\sin\theta = 0$$

3. The answer is D

$$\sin^2 x = \sqrt{3}\sin x$$

$$\sin^2 x - \sqrt{3}\sin x = 0$$

$$\sin x\left(\sin x - \sqrt{3}\right) = 0$$

$$\sin x = 0 \quad \text{or} \quad \sin x = \sqrt{3} \text{ (rejected } \because \ -1 \le \sin x \le 1)$$

$$x = 0^\circ,\ 180^\circ,\ 360^\circ$$

4. **The answer is D**

$(3\sin\theta - 1)^2 - 2$ **is maximum when** $(3\sin\theta - 1)^2$ **is maximum**

$(3\sin\theta - 1)^2$ **is maximum when** $\sin\theta = -1$

$\therefore$ **The maximum value of** $(3\sin\theta - 1)^2 - 2$

$= (3(-1) - 1)^2 - 2$

$= (-4)^2 - 2$

$= 14$

5. **The answer is D**

$\sin 30^\circ + \sin 60^\circ = \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2}$

$\sin 90^\circ = 1$

$\therefore$ **I is false**

$\cos 90^\circ = 0$

$2\cos 45^\circ = 2\left(\frac{\sqrt{2}}{2}\right) = \sqrt{2}$

$\therefore$ **II is false**

$\frac{1}{2}\tan 60^\circ = \frac{1}{2}\left(\sqrt{3}\right) = \frac{\sqrt{3}}{2}$

$\tan 30^\circ = \frac{1}{\sqrt{3}}$

$\therefore$ **III is false**

6. **The answer is B**

$$\begin{aligned}\frac{1}{\sin\theta\cos\theta} - \frac{1}{\tan\theta} &= \frac{1}{\sin\theta\cos\theta} - \frac{\cos\theta}{\sin\theta} \\ &= \frac{1-\cos^2\theta}{\sin\theta\cos\theta} \\ &= \frac{\sin^2\theta}{\sin\theta\cos\theta} \\ &= \frac{\sin\theta}{\cos\theta} \\ &= \tan\theta\end{aligned}$$

7. The answer is D

$$2\cos\theta = \sqrt{3}$$
$$\cos\theta = \frac{\sqrt{3}}{2}$$
$$\theta = 330^\circ \qquad (\because 180^\circ < \theta < 360^\circ)$$

8. The answer is C.

$$2\sin^2\theta - \sin\theta - 1 = 0$$
$$(\sin\theta - 1)(2\sin\theta + 1) = 0$$
$$\sin\theta = 1 \text{ or } -\frac{1}{2}$$
$$\theta = 90^\circ,\ 210^\circ \text{ or } 330^\circ$$

9. The answer is B

$$f(2) = 8$$
$$2(3 + k) = 8$$
$$k = 1$$

10. The answer is C

$$f(-1) = 0$$
$$(-1)^{99} + 99(-1) + c = 0$$
$$-1 - 99 + c = 0$$
$$c = 100$$

11. The answer is B

$$x^3 - 8 = x^3 - 2^3$$
$$= (x - 2)(x^2 + 2x + 2^2)$$
$$= (x - 2)(x^2 + 2x + 4)$$

12. The answer is A

$f(3) = -8$

$f(-3) = 10$

Let $f(x) = (x^2 - 9)Q(x) + Ax + B$

$f(3) = 3A + B = -8$............(i)

$f(-3) = -3A + B = 10$............(ii)

Solving (i) and (ii)

$A = -3$ and $B = 1$

$\therefore$ The remainder when $f(x)$ is divided by $x^2 - 9$

is $-3x + 1$

13. The answer is A

$3 - 4 + 2x^2 - 5x^4$

The coefficient of x^4 is -5

The coefficient of x^3 is 0

The coefficient of x^2 is 2

The coefficient of x is -4

The constant term is 3

The degree of polynomial is 4

14. The answer is C

For A

$\text{R.H.S.} = -(2x - 1) = -2x + 1 \neq \text{L.H.S.}$

$\therefore$ A is not an identity

For B

$\text{L.H.S.} = (x + 2)^2 = x^2 + 4x + 4 \neq \text{R.H.S.}$

$\therefore$ B is not an identity

For C

$\text{L.H.S.} = (x + 3)(x - 3) = x^2 - 9 = \text{R.H.S.}$

$\therefore$ C is an identity

D is not an identity

15. **The answer is C**

$$\text{L.H.S.} = Ax^2 + B(x+1)(x-2)$$
$$= Ax^2 + Bx^2 - Bx - 2B$$
$$= (A+B)x^2 - Bx - 2B$$

By comparing coefficient

$$-2B = -4$$
$$B = 2$$
$$A + B = 5$$
$$A = 3$$

16. **The answer is D**

Let $f(x) = x^3 + 2kx^2 + (k^2+1)x + 2k^2$

$$f(-k) = 1$$
$$(-k)^3 + 2k(-k)^2 + (k^2+1)(-k) + 2k^2 = 1$$
$$-k^3 + 2k^3 - k^3 - k + 2k^2 = 1$$
$$2k^2 - k - 1 = 0$$
$$k = -\frac{1}{2} \quad \text{or} \quad k = 1$$

17. **The answer is A**

$$\frac{2x-1}{x-1} - \frac{x+2}{x+1} - \frac{2}{x^2-1} = \frac{(2x-1)(x+1)}{(x-1)(x+1)} - \frac{(x+2)(x-1)}{(x+1)(x-1)} - \frac{2}{x^2-1}$$
$$= \frac{2x^2 + x - 1 - (x^2 + x - 2) - 2}{x^2 - 1}$$
$$= \frac{x^2-1}{x^2-1} = 1$$

18. **The answer is A**

$$x^3 - 4x^2 + x + 6 = (x+1)(x^2 - 5x + 6)$$
$$= (x+1)(x-2)(x-3)$$

19. The answer is B

I is not an identity since L.H.S. not always equal to R.H.S.

$$\left(x+\frac{1}{x}\right)^2 = x^2+2+\frac{1}{x^2}$$

$\therefore$ II is not an identity since L.H.S. not always equal to R.H.S.

$\therefore$ III is an identity

20. The answer is A

$$f(x) = (x-1)Q(x)$$

$$f(2x+1) = (2x+1-1)Q(x) = 2xQ(x)$$

$\therefore f(2x+1)$ is divisible by x

21. The answer is D

$$16-a^2-b^2+2ab$$

$$=16-(a^2-2ab+b^2)$$

$$=4^2-(a-b)^2$$

$$=(4+a-b)(4-a+b)$$

22. The answer is D

$$x^3-64=(x-4)(x^2+4x+16)$$

$\therefore$ A can be factorize

$$9x^2-25y^2=(3x+5y)(3x-5y)$$

$\therefore$ B can be factorize

$$x^3+64=(x+4)(x^2-4x+16)$$

$\therefore$ C can be factorize

D cannot be factorize

23. The answer is D

$$(7^a)^a = 7^{a^2}$$

24. The answer is C

$$\frac{a^{n+1}+a^{n+2}}{a^n+a^{n+1}} = \frac{a^n(a+a^2)}{a^n(1+a)} = a$$

25. The answer is C

$$\sqrt[3]{\frac{ab^{-2}}{a^{-\frac{1}{2}}b^{4}}}=\left(\frac{ab^{-2}}{a^{-\frac{1}{2}}b^{4}}\right)^{\frac{1}{3}}=\left(a^{1-(-\frac{1}{2})}b^{-2-4}\right)^{\frac{1}{3}}=\left(a^{\frac{3}{2}}b^{-6}\right)^{\frac{1}{3}}=a^{\frac{1}{2}}b^{-2}=\frac{a^{\frac{1}{2}}}{b^{2}}$$

26. The answer is B

$25^{x}+125=6(5^{x+1})$

$\left(5^{x}\right)^{2}+125=30(5)^{x}$

$\left(5^{x}\right)^{2}-30(5)^{x}+125=0$

$5^{x}=5$ or $5^{x}=25$

x= 1 or 2

27. The answer is D

$$\frac{\log a^{4}}{\log\sqrt{a}}=\frac{4\log a}{0.5\log a}=8$$

28. The answer is C

29. The answer is C

30. The answer is D

$$f(\log_2 x)=\log_2(\log_2 x)-(\log_2 x)^{-1}$$

31. The answer is C

$y=(\log 100x)+1=\log 100+\log x+1=\log x+3$

So the graph of y will shift upwards 3 units

32. The answer is D

33. The answer is A

34. The answer is D

$(\log_2 x)^2 = 2 + \log_2 x$

$(\log_2 x)^2 - \log_2 x - 2 = 0$

$\log_2 x = 2$ or $\log_2 x = -1$

$x = 4$ or $x = \frac{1}{2}$

35. The answer is B

$x = \log_{10} 5$, $y = \log_{10} 3$

$\log_{10}\sqrt{\frac{27}{8}}$

$= \log_{10}\left(\frac{3}{2}\right)^{\frac{3}{2}}$

$= \frac{3}{2}(\log_{10}3 - \log_{10}2)$

$= \frac{3}{2}\left(\log_{10}3 - \log_{10}\frac{10}{2}\right)$

$= \frac{3}{2}(\log_{10}3 - \log_{10}10 + \log_{10}2)$

$= \frac{3}{2}(\log_{10}3 - 1 + \log_{10}2)$

$= \frac{3}{2}(x + y - 1)$

36. The answer is B

$\log_p k = 7 \Rightarrow p^7 = k$

37. The answer is C

$10^a = 2,\ 10^b = 3$

$a = \log 2,\ b = \log 3$

$\log\sqrt{4.5} = \frac{1}{2}\log(4.5) = \frac{1}{2}\log\frac{9}{2} = \frac{1}{2}\left(\log 3^2 - \log 2\right)$

$= \frac{1}{2}(2\log 3 - \log 2) = \frac{1}{2}(2b - a)$

38. The answer is C

$$\frac{\frac{1}{3}\log 8+\frac{1}{2}\log 0.36}{\log 3+2\log 2}=\frac{\log 8^{\frac{1}{3}}+\log 0.36^{\frac{1}{2}}}{\log 3+\log 2^2}$$

$$=\frac{\log 2+\log 0.6}{\log 3(2^2)}$$

$$=\frac{\log 1.2}{\log 12}$$

$$=\frac{\log 12-\log 10}{\log 12}$$

$$=1-\frac{1}{\log 12}$$

39. The answer is C

$$\log_{10}(a^{\log_{10}a})=\log_{10}a(\log_{10}a)=\ (\log_{10}a)^2$$

40. The answer is B

$$\log(x-4)+\log x=\log(3x-10)$$

$$\log(x-4)(x)=\log(3x-10)$$

$$\log(x^2-4x)=\log(3x-10)$$

$$x^2-4x=3x-10$$

$$x^2-7x+10=0$$

$$x=5 \text{ or } x=2(\text{rejected})$$